BULK ARRIVAL RETRIAL QUEUE WITH NON PERSISTENT CUSTOMERS, ACTIVE BREAKDOWN, DELAYED REPAIR AND ORBITAL SEARCH

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ABSTRACT

Bulk arrival general service retrial queue is analyzed. Server provides two phase of service: essential and optional. After each service completion, the server searches for customers in the orbit. Customers may balk or renege at particular times. Accidental breakdown of the server is considered. The repair of the failed server starts after a random amount of time known as delay time. After repair the server continues the service of the interrupted customer or waits for the same customer. The necessary and sufficient condition for the system to be stable is presented. By applying supplementary variable technique, the steady state distributions of the server state and the number of customers in the orbit are obtained. Numerical examples are presented to illustrate the influence of the parameters on several performance characteristics.

Key Words - Retrial Queue, Server Breakdown, Orbital Search, Delayed Repair, Reserved Time, Steady State.

INTRODUCTION

In many waiting line systems, the role of the server is played by mechanical/electronic device, such as computer, ATM, traffic light etc., which is subject to accidental random failures; until the failed server is repaired, it may cause a halt in service. Wang [14], Wang and Li [13], Atencia et al. [6], Geni Gupur [11], Aissani [5], Baskar et al.[7], and Rehab et al.[12] have studied the queueing system with random breakdown. In most of the articles with server breakdown it is assumed that, the failed service channel undergoes repair instantaneously. But in several life situations, men have to be employed who may not be available on call to affect the necessary repair. Consequently a delay is caused in starting the repair of the breakdown server.

Queueing systems with impatient customers appear in many real life situations such as those involving impatient telephone switch board customers, hospital emergency rooms handling critical patients and inventory systems that store perishable goods.

Retrial queues have been widely used to model many problems arising in telecommunication and computer networks. Retrial queueing models are characterized by the feature that arriving calls which find the server busy, do not line up or leave the system immediately forever, but go to some virtual place called as orbit and try their luck again after some random time. During the last two decades considerable attention has been paid to the analysis of queueing system with repeated calls (also called retrial queues or queues with returning customers) see for example the book by Falin and Templeton [10], Artalejo and Gomez-Corral [3] and the survey papers of Artalejo [1],[2].

The next specific feature considered in retrial queueing system is orbital search. Artalejo et al.[4], Dudin et al. [9] and Chakravarthy et al. [8] have investigated retrial queue with orbital search to reduce the idle time of the server.

In this paper, bulk arrival two phase retrial queueing systems with balking, reneging, orbital search and server breakdown is discussed by including the concept of delay time and reserved time. Analytical treatment of this model is obtained by supplimentary variable technique. The main motivation is from applications to Local Area Networks, client server communication and electronic mail services on internet.

MODEL DESCRIPTION

Consider a single server retrial queue in which customers arrive in batches of variable size according to a Poisson process with rate λ. The batch size Y is a random variable with probability mass function P(Y=k) = C_k, the probability generating function C(z) and first two moments m_1 and m_2. At the arrival epoch of a batch, if the server is idle, then one of the customers get into the service immediately and others leave the service area and enter the retrial queue. If the server is busy or down, the arriving customers either join a retrial queue with probability p or leave the system with probability 1-p. The customer at the head of the retrial queue competes with potential primary customers to decide which customer will enter service next. If a primary customer arrives first, the retrial customer may cancel its attempt for service and either return to its position in the retrial queue with probability q or quit the system with probability 1-q. The retrial time of the customer is generally distributed with distribution function A(x), Laplace Stieltjes transform A*(s) and the hazard rate function $\eta(x)$.

The server provides two phases of service essential and optional. The essential service is needed to all arriving customers. As soon as the essential service is completed, the customer may opt to leave the system with probability τ or opt for the optional service with probability 1-τ. Assume i = 1 corresponds to the essential phase and i = 2 corresponds to the optional phase.

The service times follow an arbitrary distribution with distribution function $B_i(x)$, Laplace Stieltjes transform $B_i^*(s)$, the hazard rate function $\mu_i(x)$ and the first two moments μ_{i1} and μ_{i2}. If the server becomes free after completing the essential (optional) service, then immediately the server searches for customers in the orbit (if any) with probability 1-ρ_1(1-ρ_2) or remains idle with probability $\rho_1(\rho_2)$.

Assume that the server's life time has exponential distribution with mean α_i during service. When the server breaks down, there is some delay to start the repair. The delay time distributions in both service phases are arbitrarily distributed with probability distribution functions $V_i(x)$, Laplace Stieltjes transform $V_i^*(s)$, the hazard rate function $\gamma_i(x)$ and the first two moments v_{i1}, v_{i2}. The repair time in both service phases are also generally distributed with probability distribution functions $H_i(x)$, Laplace Stieltjes transform $H_i^*(s)$, the hazard rate function $\beta_i(x)$ and the first two moments h_{i1}, h_{i2}.

When the server fails in essential (optional) service, the interrupted customer either remains in the service position with probability $r_1(r_2)$ until the server is up or leaves the service area with probability 1-r_1(1-r_2) and keeps returning at times exponentially distributed with the rate $\theta_1(\theta_2)$, until the server is repaired.

If the interrupted customer remains in service position, the server after completion of repair resumes service immediately. Otherwise the server waits for the same customer to return. This time is referred as reserved time. The server is not allowed to accept new customers until the interrupted customer leaves the system after completion of service. The server is said to be blocked if the server is busy or down.

At time t, let N(t) be the number of customers in the retrial queue, W(t) the elapsed retrial time of the customer in the retrial queue, X(t) the elapsed service time of the customer in service, U(t) the elapsed delay time in attending the failed server, Y(t) the elapsed repair time of the server and R(t) the elapsed reserved time. Define the following state probabilities:

$I_0(t)$ is the probability that the server is idle at time t, and there is no customer in the retrial orbit.

$I_n(t, w)\,dw$ is the joint probability that at time t there are n customers in the orbit, the server is idle and the elapsed retrial time of an orbital customer is between w and w + dw, $n \geq 1$.

$P_n^{(i)}(t,x)dx$ is the joint probability that at time t there are n customers in the retrial orbit, the server is providing service with the elapsed service time between x and x + dx, where $n \geq 0$.

$D_{0,n}^{(i)}(t,x,u)dxdu$ $(D_{1,n}^{(i)}(t,x,u)dxdu)$ is the joint probability that at time t there are n customers in the retrial orbit, the server is down, the elapsed service time is x, the interrupted customer remains in the service position (not in the service position) and the elapsed delay time is between u and u + du, where $n \geq 0$.

$F_{0,n}^{(i)}(t,x,y)dxdy$ $(F_{1,n}^{(i)}(t,x,y)dxdy)$ is the joint probability that at time t there are n customers in the retrial orbit, the server is down, the elapsed service time is x, the interrupted customer remains in the service position (not in the service position) and the elapsed repair time is between y and y + dy, where $n \geq 0$.

$R_n^{(i)}(t,x,r)dxdr$ is the joint probability that at time t there are n customers in the retrial orbit, the elapsed service time is equal to x and the elapsed reserve time is between r and $r + dr$, where $n \geq 0$.

Theorem

The necessary and sufficient condition for the system is to be stable is

$$\lambda p m_1 [\mu_{11}(1+\alpha_1 \frac{1-r_1}{\theta_1} + h_{11}+v_{11}))+(1-\tau)\ \mu_{21}(1+\alpha_2(\frac{1-r_2}{\theta_2} + h_{21}+v_{21}))] < 1 - (1-A^*(\lambda))\ (q+m_1-1)\ (\tau\ \rho_1+\rho_2(1-\tau))$$

Proof

Let $S^{(k)}$ be the generalized service time of the k^{th} customer in service. Then $\{S^{(k)}\}$ are independently and identically distributed with Laplace transform

$$B^*(s)=B_1^*(s+\alpha_1-\alpha_1(\frac{r_1 s + \theta_1}{s + \theta_1} H_1^*(s)V_1^*(s)))+(1-\tau)B_2^*(s+ \alpha_2 - \alpha_2 (\frac{r_2 s + \theta_2}{s + \theta_2} H_2^*(s)\ V_2^*(s)))$$

and expected value

$$E(S^{(k)}) = \mu_{11}(1+ \alpha_1(\frac{1 - r_1}{\theta_1} + h_{11} + v_{11})) + (1 - \tau)\ \mu_{21}(1 + \alpha_2(\frac{1 - r_2}{\theta_2} + h_{21} + v_{21}))$$

Suppose the retrial queue has a large number of customers in the following discussion. Let P(S) and P(I) denote respectively the probabilities that the system is blocked and idle. Let $E(S^{(k)})$ be the expected blocked time and E(I) be the expected idle time. Then

$$P(S) = \frac{E(S^{(k)})}{E(S^{(k)}) + E(I)} \quad \text{and} \quad P(I) = \frac{E(I)}{E(S^{(k)}) + E(I)}$$

The arrival rate at the retrial queue when the system is blocked is $\lambda\ p m_1 P(S)$.

The arrival rate at the retrial queue when the server is idle is

$(1 - A^*(\lambda))\ (m_1 - 1)\ (\rho_1\ \tau + (1 - \tau)\ \rho_2)\ P(I) / E(I)$.

Total arrival rate at the retrial queue is

$\lambda\ p m_1 P(S) + (1 - A^*(\lambda))\ (m_1 - 1)\ (\rho_1\ \tau + (1 - \tau)\ \rho_2)\ P(I) / E(I)$.

The exit rate from the retrial queue by entering service when there is no orbital search is

$A^*(\lambda))\ (\rho_1\ \tau + (1 - \tau)\ \rho_2)\ P(I) / E(I)$.

The exit rate from the retrial queue by the orbital search is

$(\tau\ (1 - \rho_1) + (1 - \tau)\ (1 - \rho_2))\ P(I) / E(I)$.

The exit rate from the retrial queue by leaving the system when a primary customer arrives first at the server is

$(1 - q)\ (1 - A^*(\lambda))\ (\rho_1\ \tau + (1 - \tau)\ \rho_2)\ P(I) / E(I)$.

The total exit rate from the retrial queue is

$[1 - q\ (1 - A^*(\lambda))\ (\rho_1\ \tau + (1 - \tau)\ \rho_2)]\ P(I) / E(I)$.

For stability, the arrival rate should be less than the exit rate. Hence

$\lambda\ p m_1 P(S) + (1-A^*(\lambda))(m_1-1)\ (\rho_1\tau+(1-\tau)\ \rho_2)\ P(I) / E(I) < [1 - q\ (1 - A^*(\lambda))\ (\rho_1\tau+(1- \tau)\ \rho_2)]\ P(I) / E(I)$.

and hence

$$\lambda\ p m_1\ [\mu_{11}(1+\alpha_1(\frac{1 - r_1}{\theta_1} + h_{11}+v_{11}))+(1 - \tau)\ \mu_{21}(1 + \alpha_2(\frac{1 - r_2}{\theta_2} + h_{21} + v_{21}))] < 1 - (1 - A^*(\lambda))\ (q + m_1 - 1)\ (\rho_1\ \tau + (1 - \tau)\ \rho_2).$$

STEADY STATE DISTRIBUTIONS

The system of equations that governs the model under steady state, by supplementary variable method are

$$\lambda I_0 = \tau\int_0^\infty p_0^{(1)}\mu_1(x)dx + \int_0^\infty p_0^{(2)}\mu_2(x)dx \tag{1}$$

$$\frac{dI_n(w)}{dw} = -(\lambda+\eta(w))I_n(w),\ \mathrm{n}\geq 1 \tag{2}$$

$$\frac{dP_n^{(i)}(x)}{dx} = -(p\lambda+\mu_i(x)+\alpha_i)P_n^{(i)}(x) + \int_0^\infty F_{0,n}^{(i)}(x,y)\beta_i(y)dy + \theta_i\int_0^\infty R_n^{(i)}(x,r)dr + p\lambda(1-\delta_{0n})\sum_{k=1}^{n} c_k P_{n-k}^{(i)}(x),\quad n\geq 0, \mathrm{i}=1,2 \tag{3}$$

$$\frac{\partial D_{j,n}^{(i)}(x,u)}{du} = -(p\lambda+\gamma_i(u))D_{j,n}^{(i)}(x,u) + p\lambda(1-\delta_{0n})\sum_{k=1}^{n} c_k D_{j,n}^{(i)}(x,u), n\geq 0,\ i=1,2;\ \mathrm{j}=0,1 \tag{4}$$

$$\frac{\partial F_{j,n}^{(i)}(x,y)}{dy} = -(p\lambda+\beta_i(u))F_{j,n}^{(i)}(x,y) + p\lambda(1-\delta_{0n})\sum_{k=1}^{n} c_k F_{j,n}^{(i)}(x,y), n\geq 0,\ i=1,2;\ \mathrm{j}=0,1 \tag{5}$$

$$\frac{\partial R_n^{(i)}(x,r)}{dr} = -(p\lambda+\theta_i)R_n^{(i)}(x,r) + p\lambda(1-\delta_{0n})\sum_{k=1}^{n} c_k R_n^{(i)}(x,r), n\geq 0,\ i=1,2 \tag{6}$$

The steady state boundary conditions are

$$I_n(0) = \tau\rho_1\int_0^\infty p_n^{(1)}(x)\mu_1(x)dx + \rho_2\int_0^\infty p_n^{(2)}(x)\mu_1(x)dx, \mathrm{n}\geq 1 \tag{7}$$

$$P_n^{(1)}(0) = \tau(1-\rho_1)\int_0^\infty p_{n+1}^{(1)}(x)\mu_1(x)dx + (1-\rho_2)\int_0^\infty p_{n+1}^{(2)}(x)\mu_2(x)dx + \lambda C_{n+1}I_0 + \lambda(1-q)\sum_{k=1}^{n+1} C_k\int_0^\infty I_{n-k+2}(w)dw + \int_0^\infty I_{n+1}(w)\eta(w)dw + \lambda q(1-\delta_{0n})\sum_{k=1}^{n} C_k\int_0^\infty I_{n-k+1}(w)dw, \mathrm{n}\geq 1 \tag{8}$$

$$P_n^{(2)}(0) = (1-\tau)\int_0^\infty p_n^{(1)}(x)\mu_1(x)dx,\quad \mathrm{n}\geq 0 \tag{9}$$

$$D_{0,n}^{(i)}(x,0) = r_i\alpha_i P_n^{(i)}(x), n\geq 0, i=1,2 \tag{10}$$

$$D_{1,n}^{(i)}(x,0) = (1-r_i)\alpha_i P_n^{(i)}(x), n\geq 0, i=1,2 \tag{11}$$

$$F_{j,n}^{(i)}(x,0) = \int_0^\infty D_{j,n}^{(i)}(x,u)\gamma_i(u)du, n\geq 0, i=1,2,\ \mathrm{j}=0,1 \tag{12}$$

$$R_n^{(i)}(x,0) = \int_0^\infty F_{i,n}^{(i)}(x,y)\beta_i(y)dy, n\geq 0, i=1,2 \tag{13}$$

Define the following probability generating functions

$$I(z,w)=\sum_{n=1}^{\infty} I_n(w)z^n,\quad \mathrm{P}^{(i)}(z,x)=\sum_{n=0}^{\infty} P_n^{(i)}(x)z^n,\quad \mathrm{D}_j^{(i)}(z,x,u)=\sum_{n=o}^{\infty} D_{j,n}^{(i)}(x,u)z^n,$$

$$\mathrm{F}_j^{(i)}(z,x,y)=\sum_{n=o}^{\infty} F_{j,n}^{(i)}(x,y)z^n,\quad \mathrm{R}^{(i)}(z,x,r)=\sum_{n=0}^{\infty} R_n^{(i)}(x,r)z^n,\ \mathrm{i}=1,2;\ \mathrm{j}=0,1 \text{ and } |\mathrm{z}|\le 1 \qquad (14)$$

Proceeding in the usual manner with the equations (1) – (13) we can determine the expressions of $I(z,w),\ \mathrm{P}^{(i)}(z,x),\ \mathrm{D}_j^{(i)}(z,x,u),\ \mathrm{F}_j^{(i)}(z,x,y)$ and $\mathrm{R}^{(i)}(z,x,r)$, i = 1,2; j = 1,2.

By defining the the partial probability generating function $\psi(z)=\int_0^{\infty}\psi(\mathrm{z,x})\,\mathrm{dx}$ for $\psi(z,x)$ and $\psi(z)=\int_0^{\infty}\int_0^{\infty}\psi(\mathrm{z,x,y})\,\mathrm{dx\,dy}$ for $\psi(z,x,y)$, we can find the queue size probability generating functions $I(z),\ \mathrm{P}^{(i)}(z),\ \mathrm{D}_j^{(i)}(z),\ \mathrm{F}_j^{(i)}(z)$ and $\mathrm{R}^{(i)}(z)$.

PERFORMANCE MEASURES

Taking z = 1 in $I(z),\ \mathrm{P}^{(i)}(z),\ \mathrm{D}_j^{(i)}(z),\ \mathrm{F}_j^{(i)}(z)$ and $\mathrm{R}^{(i)}(z)$ we obtain the following system state probabilities

The steady state probability that the system is non-empty and the server is idle is

$$\mathrm{I} = \mathrm{I}(1) = I_0\,(1-A^*(\lambda))[K_1'(1)+(1-\tau)K_2'(1) + m_1(\tau\rho_1+(1-\tau)\rho_2)-1] / T_1 \qquad (15)$$

The steady state probability that the server is busy is

$$\mathrm{P} = P^{(1)}(1)+P^{(2)}(1)$$

$$= I_0\,\lambda N_2 [\mu_{11} + (1-\tau)\mu_{21}] / T_1 \qquad (16)$$

The steady state probability that the breakdown server waits for its repair to start is given by

$$\mathrm{D} = D_0^{(1)}(1)+D_1^{(1)}(1)+D_0^{(2)}(1)+D_0^{(2)}(1)$$

$$= I_0\,\lambda N_2 [\alpha_1 \mu_{11} v_{11} + (1-\tau)\alpha_2 \mu_{21} v_{21}] / T_1 \qquad (17)$$

The steady state probability that the server is under repair is

$$\mathrm{F} = F_0^{(1)}(1)+F_1^{(1)}(1)+F_0^{(2)}(1)+F_0^{(2)}(1)$$

$$= I_0\,\lambda N_2 [\alpha_1 \mu_{11} h_{11} + (1-\tau)\alpha_2 \mu_{21} h_{21}] / T_1 \qquad (18)$$

The steady state probability that the server is in reserved time is

$$\mathrm{R} = R^{(1)}(1)+R^{(2)}(1)$$

$$= I_0\,\lambda N_2[(1-r_1)\alpha_1 \mu_{11} / \theta_1 + (1-\tau)(1-r_2)\alpha_2 \mu_{21} / \theta_2] / T_1 \qquad (19)$$

Using the normalizing condition $I_0+\mathrm{I}+\mathrm{P}+\mathrm{D}+\mathrm{F}+\mathrm{R}=1$ the expression of I_0 is derived as $I_0 = T_1 / N_1$

(20)

where

$T_1 = 1 - K_1'(1)-(1-\tau)K_2'(1)-(1-A^*(\lambda))(q+m_1-1)(\tau\rho_1+\rho_2(1-\tau))$

$N_1 = A^*(\lambda)+(1-q)(1-A^*(\lambda))(\tau\rho_1+\rho_2(1-\tau))+(K_1'(1)+(1-\tau)K_2'(1))(A^*(\lambda)m_1(1-p)+(1-q)(1-A^*(\lambda)))/(pm_1)$

$N_2 = m_1 A^*(\lambda) + (1-q)(1-A^*(\lambda))$

$K_i'(1) = p\lambda\, \mu_{i1}\, m_1 (1 + \alpha_i (\frac{1-r_i}{\theta_i} + h_{i1} + v_{i1})), \quad i = 1, 2.$

The probability generating function of the number of customers in the retrial queue is given by

$$P_q(z) = I_0 + I(z) + \sum_{i=1}^{2}[P^{(i)}(z) + \sum_{j=0}^{1}[D_j^{(i)}(z) + F_j^{(i)}(z)] + R^{(i)}(z)]$$

The probability generating function of the number of customers in the system is given by

$$P_s(z) = I_0 + I(z) + z\sum_{i=1}^{2}[P^{(i)}(z) + \sum_{j=0}^{1}[D_j^{(i)}(z) + F_j^{(i)}(z)] + R^{(i)}(z)]$$

The mean number of customers in the orbit is

$$L_q = P_q'(1)$$

$$= \frac{T_1 N_4 + T_2 N_3 + T_1 N_3 m_2 / 2m_1}{pT_1 N_1 m_1} \qquad (21)$$

The mean number of customers in the system is

$$L_s = P_s'(1)$$

$$= \frac{T_1 N_5 + T_2 N_3 + T_1 N_3 m_2 / 2m_1}{pT_1 N_1 m_1} \qquad (22)$$

where

$T_2 = 1-(1-A^*(\lambda))(\tau\rho_1+\rho_2(1-\tau))[K_1'(1)(q+m_1-1)+qm_1+m_2/2]- K_1'(1)-(1-\tau) K_2'(1)[K_1'(1)+$

$(1-\rho_2)+\rho_2(A^*(\lambda)+(1-A^*(\lambda))(q+m_1))]-(K_1''(1)+(1-\tau) K_2''(1))/2$

$N_3 = pm_1(1-A^*(\lambda))[1-(1-q)(\tau \rho_1+\rho_2(1-\tau))]-[K_1'(1)+(1-\tau) K_2'(1)]\,[m_1(1-p)A^*(\lambda)+(1-q)(1- A^*(\lambda))]$

$N_4=p[\tau\rho_1+\rho_2(1-\tau)][(1-A^*(\lambda))(1-q)(m_1(q+ K_1'(1))+m_2/2)-m_1A^*(\lambda) K_1'(1)]+pA^*(\lambda)[m_1+m_2/2] +pm_1(1-\tau)[\rho_2(1-q)(1-A^*(\lambda)) K_2'(1)$

$- K_2''(1)(1-\rho_2)/2]+[K_1'(1)+(1-\tau) K_2'(1)]\,[m_1(1-q)(1-A^*(\lambda))+A^*(\lambda)(1-p)(m_1+m_2/2)]$

$+[K_1''(1)+(1-\tau) K_2''(1)+2(1-\tau) K_1'(1) K_2''(1)][N_2pm_1A^*(\lambda)]$

$N_5 = N_4 + N_2 [K_1'(1) + (1-\tau) K_2''(1)]$

$K_i''(1) = \quad p^2 \lambda^2 m_1^2 (\mu_{i2}(1 + \alpha_i (\frac{1-r_i}{\theta_i} + h_{i1} + v_{i1}))^2 + 2\alpha_i\, \mu_{i1}((\frac{1-r_i}{\theta_i})(\frac{1}{\theta_i} + h_{i1} + v_{i1})$

$+ (h_{i2} + 2h_{i1}v_{i1}+v_{i2})/2)) + p\lambda\, m_2\, \mu_{i1} (1 + \alpha_i (\frac{1-r_i}{\theta_i} + h_{i1} + v_{i1})), \quad i = 1, 2$

RELIABILITY INDEXES OF THE SERVER

We now consider some reliability quantities of the server. Let A(t) be the system availability at time t. The steady state availability of the server A is

$$\mathcal{A} = \lim_{t\to\infty} A(t).$$

$$= \lim_{t\to\infty} P\{\text{the service station is up at time t}\}$$

$$= I_0 + \sum_{n=1}^{\infty}\int_0^{\infty} I_n(w)\,dw + \sum_{n=0}^{\infty}\sum_{i=1}^{2}[\int_0^{\infty} P_n^{(i)}(x)dx + \int_0^{\infty}\int_0^{\infty} R_n^{(i)}(x,r)dxdr]$$

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$= \quad I_0 + I + P + R$

$= \{A^*(\lambda) + (1-A^*(\lambda))(1-q)\,(\tau\rho_1 + \rho_2(1-\tau)) - A^*(\lambda)\,(K_1'(1) + (1-\tau)\,K_2'(1))$

$$+\ \lambda\, N_2\,[\mu_{11}(1 + \alpha_1(\frac{1-r_1}{\theta_1})) + (1-\tau)\,\mu_{21}\,(1 + \alpha_2\,(\frac{1-r_2}{\theta_2}))]\}\,/\,N_1 \quad (23)$$

The steady state failure frequency of the server is

$$\mathcal{F} = \sum_{n=0}^{\infty}\sum_{i=1}^{2}\int_0^{\infty}\alpha_i P_n^{(i)}(x)dx$$

$$= N_2\,[\alpha_1\,\mu_{11} + \alpha_2\,(1-\tau)\,\mu_{21}]\,/\,N_1 \quad (24)$$

SPECIAL CASES

The model under study is a generalization of many queueing models.

Case 1: If C(z) = z, then the model reduces to M/G/1 two phase retrial queueing system with server breakdown, delayed repair and orbital search.

Case 2: If $\tau = 1$ and $\rho_1 = \rho_2 = 1$ the system reduces to single phase retrial queueing system with server breakdown and delayed repair.

Case 3: If $A^*(\lambda) \to 1$, p = q = $\rho_1 = \rho_2 = 1$, C(z) = z, this model reduces to M/G/1 queue with second optional service and server breakdowns with delayed repairs.

Case 4 : If $A^*(\lambda) \to 1$, p = q = $\rho_1 = \rho_2 = r_1 = r_2 = 1$, C(z) = z, the system reduces to M/G/1 queue with second optional service and server breakdowns.

Case 5: If $A^*(\lambda) \to 1$, p = q = $\rho_1 = \rho_2 = 1$, $\alpha_1 = \alpha_2 = 0$, C(z) = z, then the model reduces to An M/G/1 queue with second optional service.

NUMERICAL RESULTS

We present some numerical results to illustrate the effect of varying parameters on the performance measures of our system.

Assume that the service time, retrial time, delay time and repair time distributions are exponential with rate respectively μ_1, μ_2, η, γ_1, γ_2, β_1 and β_2. For the choice of parameters (λ, η, μ_1, μ_2, γ_1, γ_2, β_1, β_2, r_1, r_2, ρ_1, ρ_2, τ, θ_1, θ_2, C_1, C_2, α_1, α_2, p, q) = (1, 10, 10, 15, 6, 4, 5, 4, 0.5, 0.5, 0.7, 0.7, 0.5, 5, 5, 0.6, 0.4, 5, 3, 0.8, 0.8).

Tables 1 - 3 give the values of I_0, I, P and L_q.

From Tables 1 and 2, it is observed that for the increasing values of 1 – p and 1 – q, the values of I_1 and P_1 decrease whereas I_0 increases which agree with the intuitive expectations. Due to orbital search the expected orbit size increases to a certain level and then decreases. Numerical results for various values of repair rate β_2 are shown in Table 3. As expected, I_0 and P_1 increase while I_1 and L_q decrease with the increase in β_2.

Assume that the distributions of service time, retrial time, delay time and repair time follow (i) Erlangian of order 2, (ii) Exponential, (iii) Hyper-exponential distributions. Table 4 provides the availability of the server A and the failure frequency F by variying the values of μ_1, τ and α_1 for the three processes Erlang, exponential and hyper-exponential with the fixed values (λ, η, μ_2, γ_1, γ_2, β_1, β_2, r_1, r_2, ρ_1, ρ_2, θ_1, θ_2, C_1, C_2, α_2, p, q, a) = (1, 15, 20, 5, 6, 3, 6, 0.5, 0.5, 0.5, 0.5, 6, 3, 0.6, 0.4, 2, 0.8, 0.8, 0.5).

For all μ_1 and τ, as α_1 increases the availability of the server decreases and the failure frequency increases. This can be explained intuitively as follows: As failure rate α_1 increases life time decreases and hence availability decreases and failure rate increases.

1 – q	I_0	I_1	P_1	L_q

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Table 1. Performance measures by varying reneging probability

0.0	0.4880	0.0409	0.1667	1.2352
0.1	0.4918	0.0405	0.1663	1.2443
0.2	0.4956	0.0401	0.1658	1.2516
0.3	0.4992	0.0397	0.1653	1.2570
0.4	0.5028	0.0393	0.1649	1.2606
0.5	0.5063	0.0389	0.1644	1.2626
0.6	0.5098	0.0386	0.1640	1.2630
0.7	0.5132	0.0382	0.1636	1.2618
0.8	0.5165	0.0379	0.1631	1.2591
0.9	0.5198	0.0375	0.1627	1.2550

Table 2. Performance measures by varying balking probability

1 – p	I_0	I_1	P_1	L_q
0.0	0.4250	0.0565	0.1852	0.8920
0.1	0.4622	0.0478	0.1749	1.2516
0.2	0.4956	0.0401	0.1658	1.2516
0.3	0.5256	0.0331	0.1576	1.3232
0.4	0.5528	0.0268	0.1501	1.3526
0.5	0.5776	0.0211	0.1433	1.3520
0.6	0.6002	0.0159	0.1371	1.3298
0.7	0.6209	0.0111	0.1314	1.2915
0.8	0.6400	0.0066	0.1262	1.2411
0.9	0.6576	0.0025	0.1214	1.1814

Table 3. Performance measures by varying β_2

β_2	I_0	I_1	P_1	L_q
5	0.5016	0.0397	0.1660	1.2478
10	0.5138	0.0388	0.1664	1.2412
15	0.5178	0.0385	0.1666	1.2392
20	0.5199	0.0383	0.1667	1.2382
25	0.5211	0.0382	0.1667	1.2377
30	0.5219	0.0382	0.1667	1.2373
35	0.5225	0.0381	0.1668	1.2371
40	0.5229	0.0381	0.1668	1.2369
45	0.5233	0.0381	0.1668	1.2367
50	0.5236	0.0381	0.1668	1.2366

Table 4. Availability and Failure frequency

μ_1	τ	α_1	Erlang		Exponential		Hyper-exponential	
			$\mathcal{A}$	$\mathcal{F}$	$\mathcal{A}$	$\mathcal{F}$	$\mathcal{A}$	$\mathcal{F}$
25	0.0	1	0.7282	0.3480	0.9268	0.1876	0.9762	0.1006
		2	0.6313	0.4366	0.8989	0.2395	0.9665	0.1289
		3	0.5389	0.5211	0.8714	0.2907	0.6569	0.1569
		4	0.4507	0.6018	0.8442	0.3413	0.9473	0.1849
		5	0.3665	0.6780	0.8175	0.3912	0.9377	0.2127
	0.5	1	0.8041	0.2320	0.9484	0.1223	0.9831	0.0650
		2	0.7017	0.3267	0.9199	0.1754	0.9733	0.0935
		3	0.6044	0.4169	0.8919	0.2278	0.9636	0.1219
		4	0.5116	0.5028	0.8643	0.2795	0.9540	0.1501

		5	0.4231	0.5846	0.8370	0.3305	0.9443	0.1782
		1	0.8858	0.1071	0.9706	0.0551	0.9901	0.0289
		2	0.7775	0.2086	0.9416	0.1095	0.9803	0.0577
	1	3	0.6747	0.3050	0.9130	0.1631	0.9705	0.0863
		4	0.5769	0.3967	0.8848	0.2159	0.9607	0.1148
		5	0.4838	0.4840	0.8570	0.2680	0.9510	0.1432
		1	0.7440	0.3340	0.9313	0.1792	0.9779	0.0958
		2	0.6619	0.4090	0.9080	0.2227	0.9698	0.1193
	0.0	3	0.5831	0.4810	0.8849	0.2657	0.9618	0.1426
		4	0.5074	0.5503	0.8620	0.3082	0.9538	0.1658
		5	0.4345	0.6169	0.8394	0.3503	0.9459	0.1890
		1	0.8210	0.2165	0.9531	0.1136	0.9484	0.0601
		2	0.7343	0.2968	0.9292	0.1581	0.9767	0.0838
30	0.5	3	0.6512	0.3737	0.9057	0.2021	0.9686	0.1073
		4	0.5715	0.4475	0.8824	0.2455	0.9606	0.1308
		5	0.4948	0.5185	0.8594	0.2885	0.9526	0.1542
		1	0.9040	0.0900	0.9754	0.0461	0.9918	0.0240
		2	0.8123	0.1760	0.6511	0.0916	0.9836	0.0479
	1	3	0.7244	0.2584	0.6271	0.1366	0.9755	0.0716
		4	0.6402	0.3373	0.6034	0.1811	0.9674	0.0953
		5	0.5595	0.4130	0.8800	0.2251	0.9593	0.1189
		1	0.7553	0.3238	0.9346	0.1731	0.9790	0.0925
		2	0.6842	0.3889	0.9145	0.2106	0.9722	0.1125
	0.0	3	0.6155	0.4516	0.8946	0.2476	0.9653	0.1324
		4	0.5492	0.5123	0.8749	0.2844	0.9585	0.1523
		5	0.4850	0.5710	0.8554	0.3207	0.9517	0.1721
		1	0.8333	0.2053	0.9564	0.1073	0.9860	0.0567
		2	0.7581	0.2749	0.9359	0.1456	0.9790	0.0769
35	0.5	3	0.6856	0.3420	0.9156	0.1835	0.9721	0.0971
		4	0.3157	0.4067	0.8956	0.2211	0.9653	0.1171
		5	0.5481	0.4693	0.8757	0.2582	0.9584	0.1371
		1	0.9173	0.0776	0.9789	0.0396	0.9930	0.0205
		2	0.8376	0.1522	0.9580	0.0788	0.9860	0.0409
	1	3	0.7610	0.2241	0.9373	0.1176	0.9790	0.0612
		4	0.6841	0.2934	0.9168	0.1560	0.9721	0.0815
		5	0.6158	0.3602	0.8965	0.1940	0.9652	0.1017

REFERENCES

[1]. Artalejo, J.R. , ‘Accessible bibliography on retrial queues’, Mathematical and Computer Modeling, 30, pp 1-6, (1999a)

[2]. Artalejo, J. R. ,’A classified bibliography of research on retrial queues: progress in 1990-1999’,Top, 7, pp 187-211, (1999b)

[3]. Artalejo, J. R. and Gomez-Corral, A., Retrial Queueing Systems – A Computational Approach, Springer Verlag, Berlin, (2008).

[4]. Artalejo, J. R., Joshua, V. C. and Krishnamoorthy. A. , ‘An M/G/1 retrial queue with orbital search by server’, Advances in Stochastic Modelling, Notable Publications, Inc., NJ, pp 41-54, (2002).

[5]. Aissani, A. ‘An M/G/1 Retrial Queue with Negative Arrivals and Unreliable Server’, Proceeding of the World Congress On Engineering, Vol. I, June 30 – July 2, London, UK, (2010).

[6]. Atencia, I., Bouza,G. and Moreno, P., 'An $M^{[x]}$/G/1 retrial Queue with server breakdowns and constant rate of repeated attempts', Ann. Oper. Res., 157: pp 225-243, (2008).

[7]. Baskar, S., Rajalakshmi Rajagopal and Palaniammal. S., 'A Single Server M/G/1 Queue with Service Interruption under Bernoulli Schedule', Vol. 5, No. 35, pp 1697-1712, (2011).

[8]. Chakravarthy, S. R., Krishnamoorthy, A. and Joshua, V. C. 'Analysis of multi server retrial queue with search of customers from the orbit', Perform. Eval., 63(8), pp 776-798, (2006).

[9]. Dudin, A. N., Krishnamoorthy, A., Joshua, V. C. and Tsarenkov, G. V., 'Analysis of BMAP/G/1 retrial system with search of customers from the orbit', European Journal of Operations Research, 157(1), pp 169-179, (2004).

[10]. Falin, G. I. and Templeton, J. G. C. , 'Retrial Queues'. Chapman and Hall, London, (1997).

[11]. Geni Gupur , 'Analysis of the M/G/1 retrial queueing model with server breakdowns', J. Pseudo-Differ. Oper. Appl.,1: pp 313–340, (2010).

[12]. Rehab F. Khalaf, Kailash C. Madan and Cormac A. Lukas, 'An $M^{[x]}$/G/1 Queue with Bernoulli Schedule, General Vacation Times, Random Breakdowns, General Delay Times and General Repair Times', Applied Mathematical Sciences, Vol. 5, No.1 , pp 25-51(2011).

[13]. Wang, J. and Li H., 'Reliability Analysis of the retrial Queue with server breakdowns and repairs', Queueing Systems, 38, pp 363-380, (1994).

[14]. Wang, J., 'An M/G/1 Queue with Second Optional Service and Server Breakdowns', Computers and Mathematics with Applications, 47: pp 1713-1723, (2004).